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Temperature response of a single blow regenerator using axially dispersive thermal wave of finite propagation velocity – analysis and experiment

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Abstract

Transient behaviour of the regenerator bed is analysed using a hyperbolic dispersion model. Flow maldistribution and backmixing in fluids are taken into account by introducing an axial dispersion term in the energy equation which is assumed to follow Chester's hyperbolic conduction law. Danckwerts' type boundary condition is extended for the present analysis by introducing the finite propagation velocity of the dispersion wave which is assumed to originate at the entry of the bed (see, e.g. Danckwerts, P.V., 1953. Chemical Engineering science, Genie Chimique, vol. 2, pp. 1–14). The governing equations are solved using Laplace transformation technique. The inverse Laplace transformation is carried out by using a fast Fourier transform technique proposed by Crump (cf. Crump, K.S., 1976. J. Assoc. Comput Mach. 23, 89–96). Analysis shows that at a lower dispersive Peclet number which indicates higher rate of maldistribution and backmixing, the effect of propagation velocity of the dispersive wave plays a significant role in the transient response of the bed. Results reveal that the present approach is a more general one which at higher dispersion wave propagation velocity and lower dispersion coefficient approaches Schumann's classical model (cf. Schumann, T.E.W., 1929. J. Franklin Institute 208, 405–416). The present model is subsequently validated by conducting an experiment with a packed bed of stainless steel wire mesh subject to a sudden rise in the inlet fluid temperature. The results indicate a strong hyperbolic effect and the experimental technique developed can be used for determination of the dispersion coefficient and its propagation velocity.

Notation

A	convective heat transfer area (m^2)
A_f	fluid flow cross-sectional area (m^2)
A_m	cross-sectional area of the solid (m^2)
B_j	Eigenvalues of matrix
c	conduction wave propagation velocity (m/s)
c^*	dispersion wave propagation velocity (m/s)
C_m	specific heat of the solid (J/kg K)
C_p	isobaric specific heat of the fluid (J/kg K)
G	exchanger flow stream mass velocity (kg/m^2 s)
h	heat transfer coefficient (W/m^2 K)
K	thermal conductivity (W/m K)
L	length of the bed (m)
m_f	mass flow rate of fluid (kg/s)
m	mass of hold up fluid over the bed at a particular instant (kg)
M	mass of the solid matrix (kg)
NTU	number of transfer units ($=hA/m_f C_p$)
Pe	dispersive Peclet number ($=m_f C_p L/\lambda A_f$)
P_m	matrix Peclet number ($=m_f C_p L/k A_f$)
q	heat transfer by conduction in the axial direction (W/m^2)
q_x	heat transfer by dispersion in the axial direction (W/m^2)
Re	Reynolds number ($=4 r_h G/\mu$)
r_h	hydraulic radius (m)
s	transformed time variable in Laplace domain
T	temperature (K)
u	velocity of the fluid (m/s)
V	ratio of velocity of the fluid to dispersion wave propagation velocity ($2u/c^*$)
x	space coordinate (m)
X	dimensionless space coordinate ($=x/L$)
z	non-dimensional time ($=m_f C_f^\tau/MC_m$)
α	thermal diffusivity (m^2/s)
α^*	thermal diffusivity of axial dispersion ($=\lambda/\rho C_p$), (m^2/s)
θ	non-dimensional temperature ($=(T-T_{atm})/(T_{in}-T_{atm})$)
λ	axial dispersion coefficient of fluid (W/m K)
ρ	density of the fluid (kg/m^3)
τ	time (s)

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μ	viscosity (N s/m^2)
η	heat capacity ratio ($=MC_m/mC_p$)
Subscripts	
atm	at atmosphere
f	of the fluid
in	at the inlet
m	solid matrix
+	the section just after the heat exchanger inlet
−	the section just before the heat exchanger inlet

1. Introduction

Packed bed regenerators have important applications in thermal energy storage systems where energy demand fluctuates with time. The thermal energy storage systems are used for a variety of applications ranging from high-temperature applications such as gas turbine power plants to low-temperature applications such as cryogenic cooling systems. In gas turbine power plants, exhaust gas is used to store heat which is utilised for preheating air. Similarly coldness is stored for application of packed beds as regenerators in cryocoolers. Such regenerator beds undergo transients during charging and discharging and also during variations in external load. Designing proper systems to control such transients requires exact simulations of the dynamic behaviour of regenerative heat exchangers.

Schumann (1929) was the first to propose an analytical one-dimensional two phase model neglecting the axial conduction in the bed and analysed the transient behaviour of a semi-infinite packed bed initially at a uniform temperature. Riaz (1978) assumed the volumetric heat transfer coefficient to be infinite and proposed a one-dimensional single phase conductivity model to develop a closed form solution. Ali Montakhab (1979) proposed another closed form solution for convective heat transfer between the granular solids and a steady flow fluid neglecting axial conduction in solid. Beasley and Clark (1984) presented an analysis, with experimental verification for the transient response of a packed bed thermal storage unit using two-dimensional temperature field of both solid and fluid phase. Adebiyi and Chenevert (1996) reviewed the analytical one-dimensional models for the packed bed thermal energy storage systems and proposed an alternative axial conductivity model.

In all these studies, the fluid was assumed to have ‘plug flow’ and the effects such as flow maldistribution and backmixing were not considered. Roetzel and Xuan (1992), Xuan and Roetzel (1993) and Das and Roetzel (1995) proposed the dispersion model to analyse the transient behaviour of multipass shell and tube and plate heat exchangers. In their analyses, the effect of flow maldistribution, backmixing and all other deviations from the so-called ‘plug flow’ were described by introducing an axial dispersion term in the one-dimensional energy equation. These deviations from plug flow were considered to follow the Fourier law of conduction with axial dispersion coefficient, λ in the place of thermal conductivity, i.e.

$$q_x = -\lambda \frac{\partial T}{\partial x}. \tag{1}$$

This dispersion coefficient is a flow property unlike the thermal conductivity which is a property of the fluid, because it arises out of flow characteristics of the fluid.

The Fourier law of conduction assumes an infinite propagation velocity for a heat wave which does not hold true at cryogenic temperatures or high temperature applied for extremely short duration. Chester (1963) modified the Fourier law of conduction which is applicable in such conditions. Chester's modified form of heat conduction equation known as hyperbolic or non-Fourier conduction law is given by

$$q + \frac{\alpha}{c^2}\frac{\partial q}{\partial \tau} = -k\frac{\partial T}{\partial x}. \tag{2}$$

Here, α is the thermal diffusivity and c is the conduction wave propagation velocity. Based on this study, Roetzel and Das (1995) introduced a new concept of hyperbolic axial dispersion. They suggested that the dispersion in a fluid can never have a propagation velocity of the order of infinity even at normal temperature since dispersion is a macrolevel disturbance while conduction is a molecular disturbance. Hence, the dispersive flux equation based on Fourier law is to be replaced by an equation similar to Eq. (2) even at normal temperature,

$$q_x + \frac{\alpha^*}{c^{*2}}\frac{\partial q_x}{\partial \tau}, \tag{3}$$

$$\frac{D}{D\tau} = \frac{\partial}{\partial \tau} + u\frac{\partial}{\partial x},$$

α^* is the apparent thermal diffusivity due to dispersion in fluid and c^* is the dispersion wave propagation velocity. They applied this model to a single pass plate heat exchanger and analysed its transient behaviour.

As far as regenerator beds are concerned, flow maldistribution and backmixing are severe. Assumption of plug flow inside the bed cannot be considered to be a good approximation for analysis particularly in the transient regime. It is to be noted here that by this approach two new parameters, viz., the dispersion coefficient (λ) and dispersion wave propagation velocity (c^*) are introduced. However, it does not mean that these two are adjustable parameters to match the experimental output, rather they are two phenomenological parameters. The first one is the dispersion phenomenon which was observed long back by Taylor (1954) and popularly known as Taylor–Aris dispersion in the study of packed bed chemical reactors. Physically this can be seen as a disturbance to the plug flow which behaves like diffusion of species in mass transfer and a virtual conduction in fluid in heat transfer. The dispersion parameter λ is strongly dependent on the flow characteristics.

The second phenomenon is the hyperbolic nature of dispersion which was not only observed by Roetzel et al. (1998) but also by a series of studies on chemical reactors by Westerterp and his group which has been summed up by Benneker (1997). Physically this accounts for the fact that dispersion is a physical disturbance and it has to propagate with a finite speed and not instantaneously as suggested by the Fourier-like formulation.

In the present work, the transient behaviour of single blow regenerator bed is analysed taking deviations from plug flow into consideration by using this hyperbolic dispersion model. The only evidence of this logical proposition so far was by indirect comparison with numerical simulation of laminar flow through the tube by Roetzel et al. (1998). In the present work, transient experiment has been conducted with a wire mesh packed bed which can be regarded as the first direct experimental evidence of the proposition of hyperbolic dispersion in regenerative heat exchangers. The two dispersion parameters, viz. axial dispersion coefficient λ and dispersion wave propagation velocity c^*, are predicted from the experiment. The accuracy of the present model is better in the transient regime because it corrects the temperature difference between the fluid and matrix rather than the common practice of correcting the heat transfer coefficient. This essentially means that the present proposition is a two parameter model where the experimental technique used can be utilized for the determination of these two parameters related to axial dispersion. The present analysis can be applied to packed bed heaters, chemical reactors, testing of compact heat exchangers and regenerators with long charging periods like that of blast furnace Cowper stoves and helium recovery beds at cryogenic temperature.

2. Mathematical formulation

To analyse a single blow regenerator bed, the following assumptions were made in keeping with the literature available. Additional assumptions are incorporated to take care of axial dispersion and its propagation velocity.

1. Both the fluid and the matrix properties do not change with temperature.
2. Radial conduction resistance in the axial direction is negligible.
3. Conduction wave propagation velocity in the solid is infinite so that Fourier law of conduction can be applied to it.
4. Maldistribution and backmixing of fluid are considered by introducing an axial dispersion term in the energy equation.
5. Dispersion wave propagation velocity is finite with respect to fluid velocity.
6. Molecular (thermal) conductivity of the fluid is negligible compared to the axial dispersion coefficient.
7. There is no reflection of dispersive wave from the boundaries.

With the above assumptions, energy balance over a small control volume, yields the following dimensionless fluid and matrix equations, respectively:

$$\frac{Pe}{\eta}\frac{\partial\theta_{\mathrm{f}}}{\partial z}+Pe\frac{\partial\theta_{\mathrm{f}}}{\partial X}+\frac{V^2}{\eta^2}\frac{\partial^2\theta_{\mathrm{f}}}{\partial z^2}+\frac{2V^2}{\eta}\frac{\partial^2\theta_{\mathrm{f}}}{\partial z\partial X}+(V^2-1)\frac{\partial^2\theta_{\mathrm{f}}}{\partial X^2}$$
$$=Pe\cdot\mathrm{NTU}(\theta_{\mathrm{m}}-\theta_{\mathrm{f}})+\frac{\mathrm{NTU}\cdot V^2}{\eta}\frac{\partial}{\partial z}(\theta_{\mathrm{m}}-\theta_{\mathrm{f}})$$
$$+\mathrm{NTU}\cdot V^2\frac{\partial(\theta_{\mathrm{m}}-\theta_{\mathrm{f}})}{\partial X}, \quad (4)$$

$$\frac{\partial\theta_{\mathrm{m}}}{\partial z}=\frac{1}{P_{\mathrm{m}}}\frac{\partial^2\theta_{\mathrm{m}}}{\partial X^2}+\mathrm{NTU}(\theta_{\mathrm{f}}-\theta_{\mathrm{m}}). \quad (5)$$

3. Boundary conditions

It is assumed that before entry, there is no dispersion, so that there is a plug flow just before the entry to the bed and energy balance between the section just before and after entry gives the following boundary condition for the fluid,which is an extended form of Danckwerts (1953) boundary condition:

$$\theta_{\mathrm{f}}^{+}-\frac{(1-V^2)}{Pe}\frac{\partial\theta_{\mathrm{f}}^{+}}{\partial X}+\frac{V^2}{Pe\cdot\eta}\frac{\partial\theta_{\mathrm{f}}^{+}}{\partial z}=\theta_{\mathrm{f}}^{-}\quad \text{at } X=0. \quad (6)$$

At exit, the dispersion is 0 so that the boundary condition becomes

$$\frac{\partial\theta_{\mathrm{f}}}{\partial X}=0\quad \text{at } X=1. \quad (7)$$

For solid, the consideration of Fourier conduction yields

$$\frac{\partial\theta_{\mathrm{m}}}{\partial X}=0\quad \text{at } X=0 \text{ and } X=1. \quad (8)$$

4. Solution procedure

Initially, both the fluid and the matrix are considered to be at the same temperature yielding initial conditions

$$\theta_{\mathrm{f}}(X,z=0)=0,\qquad \theta_{\mathrm{m}}(X,z=0)=0. \quad (9)$$

Taking the Laplace transform, the governing Eqs. (7) and (8) can be transformed into a pair of ordinary differential equations using the initial conditions (Eq. (9)), and this pair of PDEs can be presented by the matrix equation

$$\frac{\mathrm{d}\bar{\theta}}{\mathrm{d}X}=\bar{A}\bar{\theta}, \quad (10)$$

where

$$\bar{\theta}=\begin{bmatrix}\theta_{\mathrm{f}}(s)\\ \mathrm{d}\theta_{\mathrm{f}}(s)/\mathrm{d}X\\ \theta_{\mathrm{m}}(s)\\ \mathrm{d}\theta_{\mathrm{m}}(s)/\mathrm{d}X\end{bmatrix}. \quad (11)$$

Similarly, the boundary conditions given by Eq. (6) can be transformed to

$$\theta_{\mathrm{f}}(s)-\frac{(1-V^2)}{Pe}\frac{\mathrm{d}\theta_{\mathrm{f}}(s)}{\mathrm{d}X}+\frac{V^2}{Pe\cdot\eta}s\theta_{\mathrm{f}}(s)=F(s)\quad \text{at } X=0 \quad (12)$$

and $\bar{A}$ is a 4×4 coefficient matrix having the following elements:

$$A_{12}=A_{34}=1,$$

$$A_{21}=\frac{((Pe\cdot s)/\eta)+((V^2s^2)/\eta^2)+Pe\cdot\mathrm{NTU}+((\mathrm{NTU}\cdot V^2s)/\eta)}{(1-V^2)},$$

$$A_{22}=\frac{Pe+((2V^2s)/\eta)+\mathrm{NTU}\cdot V^2}{(1-V^2)},$$

$$A_{23}=\frac{-Pe\cdot\mathrm{NTU}-((\mathrm{NTU}\cdot V^2s)/\eta)}{(1-V^2)},$$

$$A_{24}=\frac{-\mathrm{NTU}\cdot V^2}{(1-V^2)},$$

$$A_{41}=-\mathrm{NTU}\cdot P_{\mathrm{m}},\quad A_{43}=(\mathrm{NTU}+S)\cdot P_{\mathrm{m}}.$$

The other elements are 0.

Eq. (10) can be solved by computing the eigenvalues (B_j) and eigenvectors (u_{ij}) of the matrix. Thus, the fluid temperature, the matrix temperature and their derivatives in the frequency domain can be written as

$$\theta_{\mathrm{f}}(s)=\sum_{j=1}^{4}d_ju_{1j}\,\mathrm{e}^{B_jX}, \quad (13)$$

$$\theta_{\mathrm{m}}(s)=\sum_{j=1}^{4}d_ju_{3j}\,\mathrm{e}^{B_jX}, \quad (14)$$

$$\frac{\mathrm{d}\theta_{\mathrm{f}}(s)}{\mathrm{d}X}=\sum_{j=1}^{4}d_ju_{2j}\,\mathrm{e}^{B_jX}, \quad (15)$$

$$\frac{d\theta_m(s)}{dX} = \sum_{j=1}^{4} d_j u_{4j} e^{B_j X}, \tag{16}$$

where d_j can be found out by applying Eqs. (13)–(16) to the boundary conditions which gives the system of linear equations

$$D = W^{-1}H, \tag{17}$$

where W is a 4×4 matrix which is given by the boundary conditions. D and H are given by

$$D = [d_1, d_2, d_3, d_4]^T, \tag{18}$$

$$H = [F(s), 0, 0, 0]^T. \tag{19}$$

This procedure obviously fails in case of multiple eigenvalues. In such situations a small number is added to the eigenvalues to make them distinct. This number is chosen in such a way that the temperature field is not significantly affected by it.

The solution obtained using the above method which is in the frequency domain should be inverted back to the time domain. Obviously, it should be done using a numerical method. In the present work, a method suggested by Crump (1976) is used. In this method inversion is carried out by using a Fourier series approximation. For a function $G(s)$, the nth time step in the time domain is of the form

$$g(z_n) = \frac{\exp(az_n)}{z}\left[Re\sum_{k=0}^{M-1} G\left(a + \frac{ik\pi}{z}\right)\exp\left(i\frac{2\pi nk}{M}\right) - \frac{1}{2}G(a)\right]. \tag{20}$$

For minimizing the truncation error the constant a lies in the domain $4 < az < 5$.

5. Experimental validation

5.1. Test setup

To verify the present model, an experiment was conducted using a setup which is shown schematically in Fig. 1. Air from a blower of 15 HP capacity is heated externally by using an electric heater and a burner which burns LPG. Temperature measurements at the inlet and the outlet of the fluid are done by using K-type thermocouple. Diameter of the thermocouple is 1 mm. For accurate transient temperature measurement data acquisition A/D card with counter and cold junction compensation along with appropriate software has been used. Temperature of the heated air is measured and the air bypassed through a bypass valve till the temperature becomes steady. Air flow through the test section is controlled by using a butterfly valve. The butterfly valve is opened whenever the air reaches a steady-state temperature. Thus, a sudden change at the inlet temperature of the fluid has been achieved. The response has been observed by recording the inlet and outlet temperatures simultaneously.

The test section consists of two flanges between which is astainless wire mesh (16×16, 10×10 or 5×5) closely packed over a length of 40 mm inside a 100 mm diameter galvanized iron tube. The temperature of air at the inlet and outlet of the bed are measured. The flow rate is measured using a gas flowmeter.

5.2. Data reduction and error estimate

It is impossible to achieve a step input since the valve takes some time to open. During the experiment the recorded inlet temperature history is reduced to non-dimensional form and a first-order equation of the form

$$\theta_{f\,in} = 1 - \exp(\Phi + \psi z) \quad (\text{where } \Phi \text{ and } \psi \text{ are constants})$$

is fitted to it. In the subsequent computation, this real input function has been used instead of an approximated step function. Fig. 2 shows a typical inlet temperature curve in dimensional form, which is used for computation by subsequent non-dimensionalization and its temperature response at the exit of the bed at the air flow rate indicated in the figure. This figure indicates the nature of the transient process in terms of absolute variables of the temperature in °C and time in seconds.

In the measurement, the main source of error has been the thermocouple response delay and accuracy and the error due to flow measurement by gas flow meter. It was found that the

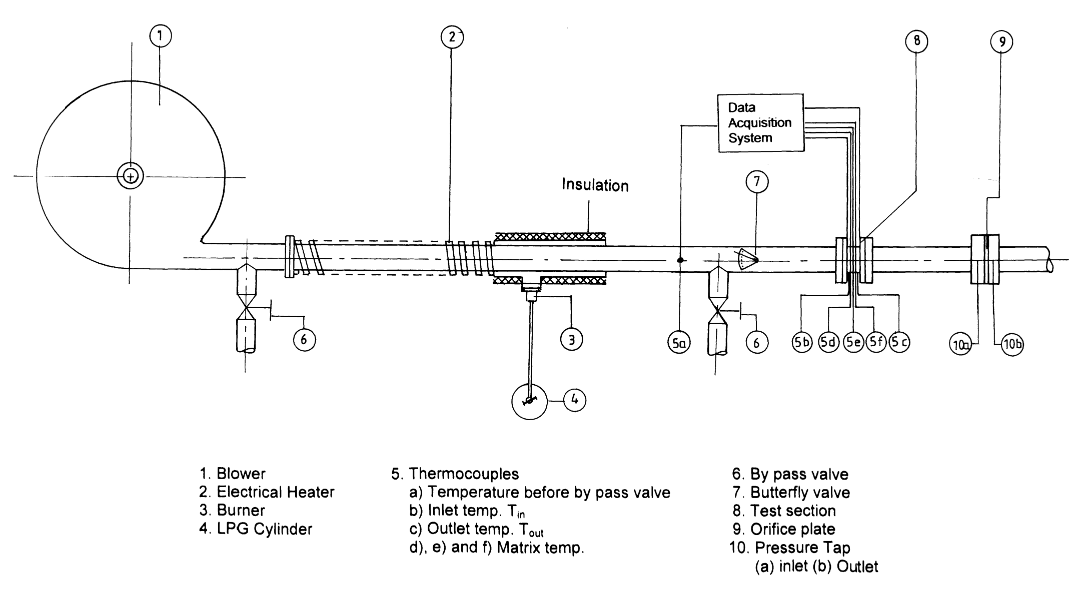

Fig. 1. Schemetic diagram of the experimental setup.

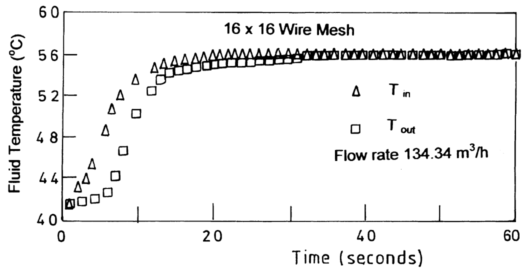

Fig. 2. Dimensional input–output temperature data of a typical response.

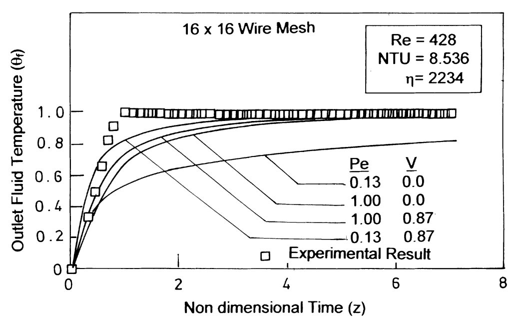

Fig. 3. Comparison of experimental and computed transient response of the packed bed (16 × 16 wire mesh, $Re = 428$).

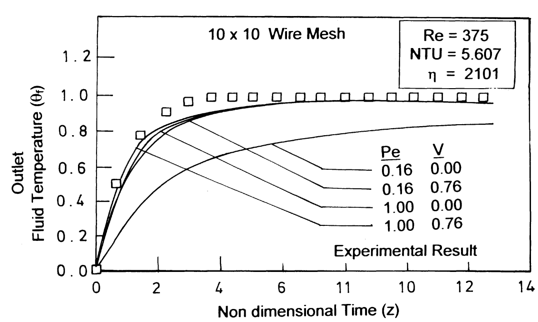

Fig. 4. Comparison of the experimental output with Schumann's model (10 × 10 wire mesh, $Re = 375$).

data acquisition unit can handle thermocouple responses with an accuracy of 0.1°C which is about 0.7% of the temperature difference achieved. The time constant of the thermocouple is 50 ms (Philips thermocoax thermocouple of cover diameter 1 mm is used) which is much less than the measurement time. It has been estimated that the error in flow measurement is limited to 2.69%. The sensitivity of the temperature response to the error in flow measurement is 3.3%, which makes the present level of accuracy acceptable.

5.3. Results

Even though the hyperbolic dispersion concept was constructed on logical foundation by Roetzel and Das (1995), the aforementioned study (or as a matter of fact the whole proposition) requires further verification for acceptance. A first effort was made by Roetzel et al. (1998) by comparing the hyperbolic model with a laminar flow through a circular tube which clearly indicates the wave nature of dispersion. To reinforce the evidence for more complex flow configuration, experiments were carried out in the present work using a stainless steel wire mesh to form the regenerator bed. The results of two such transient responses are plotted in Figs. 3 and 4 which show the fluid outlet temperature at $Re = 428$ (16 × 16 wire mesh) and $Re = 375$ (10 × 10 wire mesh). Here, Reynolds number is defined on the basis of hydraulic diameter as

$$Re = 4r_h G/\mu.$$

It is evident that the parabolic dispersion model with $V = 0$ fails totally to predict the behaviour. The heat transfer parameter NTU is taken from Keys and London (1984). A reasonable match between computation and experiment can be achieved at $Pe = 0.13$ and $V = 0.87$ for Fig. 3 and $Pe = 1.0$ and $V = 0.76$ for Fig. 4, which not only indicates a strong dispersion but also a dispersion propagation velocity of the order of fluid velocity which was also an observation of the numerical study by Roetzel et al. (1998). The importance of dispersion with finite propagation velocity can be better understood by comparing the present model with Shumann's (1929) classical plug flow model in the light of experimental observation as shown in Figs. 5 and 6. The figures clearly indicate that the proposed model describes the transient regime better than the plug flow model which reconfirms the validity of the proposition. The quality of the match can be better understood from Fig. 7, in which for the 5 × 5 wire mesh, the slope of the temperature response is compared to the theoretical impulse response which is the derivative of the step response for a linear system. This brings out the importance of incorporating wave nature of dispersion propagation particularly in the transient regime which is becoming more important due to requirement of precision process control in modern chemical industry.

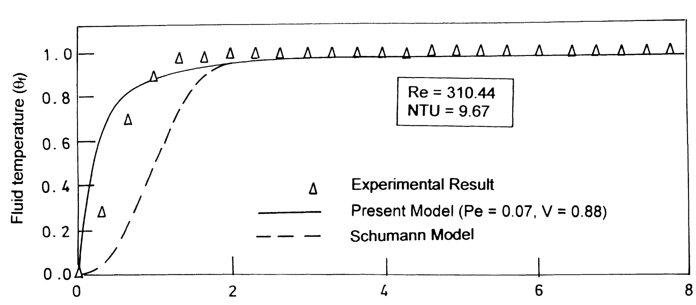

Fig. 5. Comparison of the experimental output with Schumann's model (16 × 16 wire mesh, $Re = 310.44$).

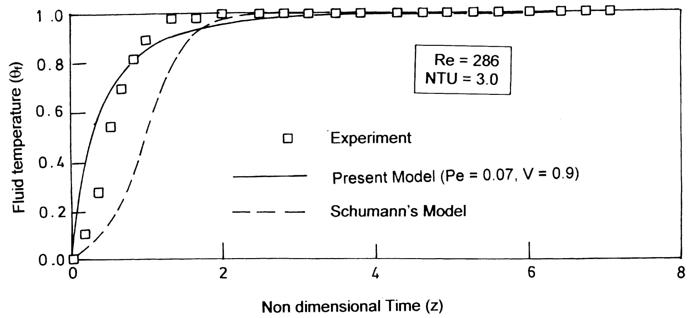

Fig. 6. Comparison of the experimental output with Schumann's model (10 × 10 wire mesh, $Re = 286$).

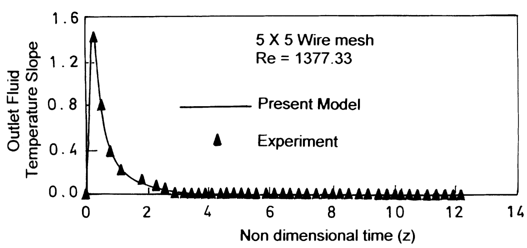

Fig. 7. Comparison between slope of the step response and the impulse response.

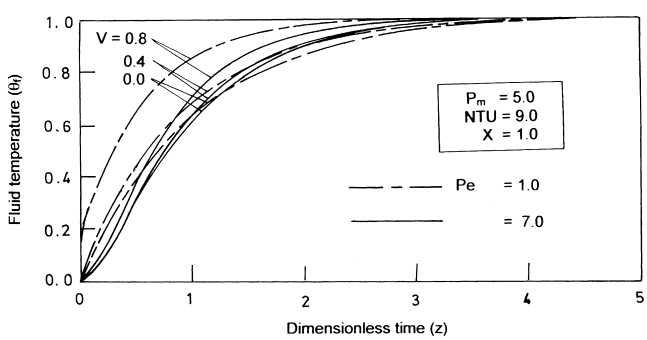

Fig. 8. Effect of dispersive *Pe* and wave propagation velocity of dispersion on the exit fluid temperature for step input (NTU = 9.0 and $P_m = 5.0$).

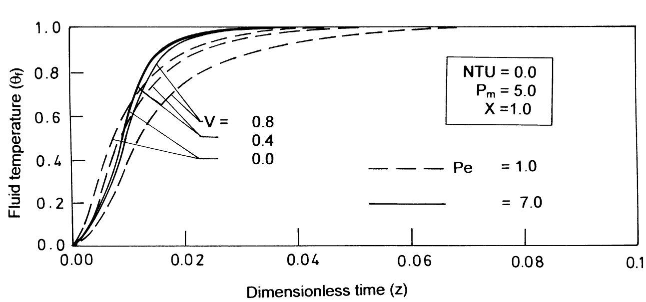

Fig. 9. Effect of dispersive *Pe* and propagation velocity of dispersion on the exit fluid temperature for step input (NTU = 0.0, $P_m = 5.0$).

6. Parametric study

In the present analysis, the emphasis has been given to depicting the effect of dispersion parameters rather than well-established heat transfer parameters such as NTU and the heat capacity ratio η. Both the temporal and spatial behaviours for a regenerator bed have been studied to bring out the essence of the present analysis. In the entire parametric study, the heat capacity ratio (η) is taken as 100. However, for comparison with experiment the actual values of η are used which are indicated in the figures.

Fig. 8 depicts the simultaneous effects of the dispersive Peclet number and ratio of the fluid velocity to dispersive wave propagation velocity V, on the fluid temperature at the exit of the bed. The curves indicate that the response and its gradient in the transient region depend critically on the dispersion wave propagation velocity. It is found that the effects of this wave propagation velocity increase with the decrease of dispersive Peclet number. This suggests that at higher level of axial dispersion, the propagation velocity of the wave plays a major role in the transient behaviour of the bed.

It is also observed that initially, fluid temperature is higher for lower dispersive Peclet number, but after some time the temperature gradient in the axial direction decreases at a lower Peclet number compared to that with a higher Peclet number. So the rate of increase in fluid temperature decreases. This is why crossover of the curves is observed. Whether these two effects due to dispersion and due to temperature gradient can compensate to show a crossover, depends on the propagation velocity of dispersion. However, it is difficult to recognise the reason for such behaviour from Fig. 8 due to the combined complex effects of changing temperature gradient in fluid and matrix and resulting variation in dispersion and diffusive flux

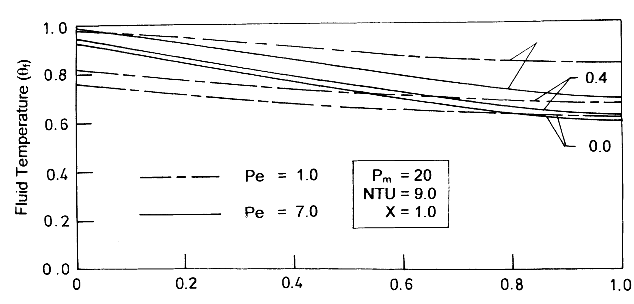

Fig. 10. Effect of P_m on spatial variation of matrix temperature for different dimensionless time for step input ($Pe = 5.0$, NTU = 9.0, $V = 0.5$).

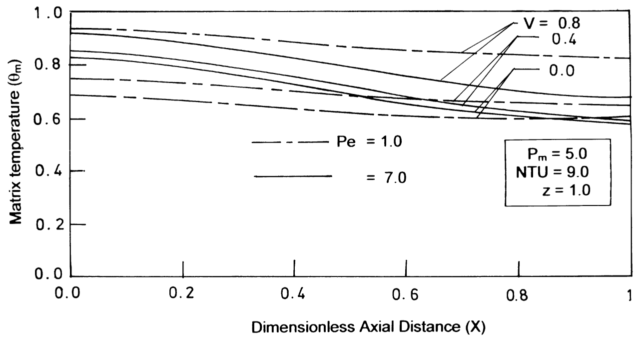

Fig. 11. Effect of dispersive *Pe* and wave propagation velocity of dispersion on the spatial variation of matrix temperature for step input ($P_m = 5.0$, $NTU = 9.0$, $z = 1.0$).

as well as the changing temperature difference between fluid and matrix resulting in a variation in convective flux. For this reason, computation has been carried out for NTU = 0.0 in order to isolate the dispersion effect from convection (this is analogous to the dispersion of a mass concentration profile injected at the inlet of a heat exchanger). The results are depicted in Fig. 9 which shows that the lower the value of *V*, the higher the dispersion effect is.

The spatial distribution of fluid and matrix temperature have been plotted from Figs. 10 and 11. It is important to note that the temperature drop at the entry of the fluid is considerably influenced by the propagation of dispersive wave. The normal parabolic dispersion studied earlier by Roetzel and Xuan (1992), Xuan and Roetzel (1993) and Das and Roetzel (1995) suggest constant temperature drop for a given Peclet number which is known to be Danckwerts' (1953) boundary condition. In the present case, it is found to change with the propagation velocity of dispersive wave as evident from Fig. 10. It is also observed that the effect of *V* on the temperature drop at entry is more at lower *Pe* as can be observed in Fig. 10. Axial distribution of the matrix temperature is also found to be influenced in a similar fashion as depicted in the figure.

Fig. 10 also reveals the effect of dispersion on the temperature profile. For $Pe = 1.0$, as the dispersion is higher, the curve is flatter. But, for $Pe = 7.0$, the curve is rather steep because of lower level of dispersion. At higher *V* the heat is transferred in the axial direction with a delay so that matrix gets heated at faster rate. Consequently, the higher the value of *V*, the higher the temperature is. The same effect prevails in the matrix temperature gradient along the bed, which is evident from Fig. 11.

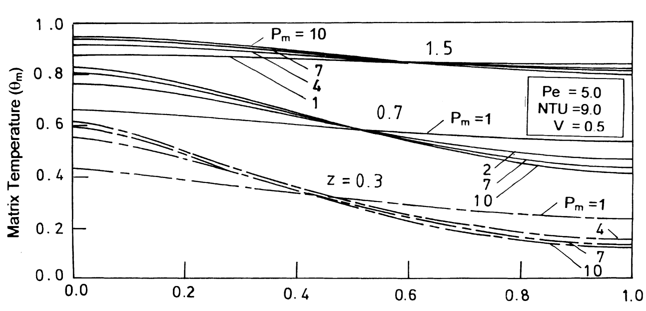

Fig. 12. Effect of P_m on spatial variation of matrix temperature for different dimensionless time for step input ($Pe = 5.0$, $NTU = 9.0$, $V = 0.5$).

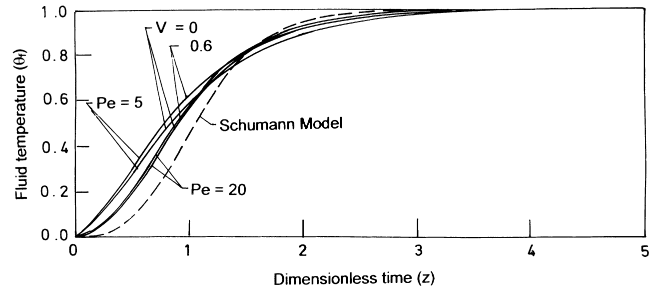

Fig. 13. Comparison of present model with Schumann's model.

The matrix temperature profiles at different levels of matrix conduction have been depicted in Fig. 12. It has been found that a higher conduction in the matrix ensures flatter spatial distribution of temperature throughout the blow period.

To assess the present analysis qualitatively the most classical model of packed bed analysis from Schumann (1929) has been compared with the present work in Fig. 13. It reveals that at lower thermal conductivity of the matrix and lower dispersion in fluid, our present model approaches Schumann's (1929) classical model since this model assumes absence of both conduction in solid and dispersion in fluid.

7. Conclusions

The hyperbolic dispersion model is invoked to analyse the transient behaviour of single blow regenerator beds. Any deviation from plug flow is taken into account by introducing an axial dispersion term in energy equation which is assumed to follow hyperbolic conduction law which indicates the propagation of a thermal wave. Solid conductivity in axial direction is considered to follow the usual Fourier's parabolic law of conduction. This brings out two important phenomena of axial dispersion and its wave-like propagation which combined take care of the deviation from the so-called plug flow.

Based on this formulation, the governing equations are formulated both for the fluid and the matrix. Danckwerts (1953) type boundary condition for dispersive systems is extended for the present analysis. Laplace transform technique is used to solve the coupled equation and for inverse Laplace transformation, a numerical technique based on fast Fourier transformation has been used.

Analysis shows that the dispersive wave propagation velocity plays an important role at lower values of Peclet number. It also shows that at low values of Peclet number, both the fluid and matrix temperature distributions over the length of the bed flatten because of higher axial dispersion. Similarly, increase of matrix conductivity also plays a significant role in distributing the temperature over the length. With the assumption of absence of conductivity in the matrix and the axial dispersion, the present model shrinks to the classical model of Schumann (1929).

Experimental result reveals that the present model is a good approximation to the transient thermal behaviour of a regenerative packed bed energy reservoir. This in fact gives a direct experimental evidence of a finite propagation velocity of the dispersive wave. It is felt that a more extensive experiment should be carried out in future for the determination of dispersion parameters like *Pe* and *V* at different Reynolds number and flow geometries in order to bring out readily usable correlations for which the present experimental method can act as a guideline.

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